Journal of Thermophysics and Heat Transfer, Vol. 6, No. 3, 1992, pp. 571–574.

[2]Brenan, P. J., and Kroliczek, E. J., *Heat Pipe Design Handbook, Vol. II*, B&K Engineering, Inc., Towson, MD, June 1979.

[3]Bowman, W. J., and Hitchcock, J., "Friction Coefficients for Flow in Pipes with Mass Injection and Extraction," *Journal of Thermophysics and Heat Transfer*, Vol. 3, No. 1, 1989, pp. 92–94.

[4]Isaacson, E., and Keller, H. B., *Analysis of Numerical Methods*, Wiley, New York, 1966.

Reference Temperature Method and Reynolds Analogy for Chemically Reacting Nonequilibrium Flowfields

David J. Debrestian* and John D. Anderson Jr.†
University of Maryland,
College Park, Maryland 20742

Introduction

INTEREST in the development of a single-stage-to-orbit vehicle (SSTO) such as the National Aerospace Plane, and other advanced hypersonic vehicles such as waveriders,[1,2] has lent impetus to the need for more accurate engineering predictions for skin friction and heat transfer on such vehicles. These vehicles can be expected to operate in a flight regime (Mach: 5–25, altitude: 15–75 km), wherein nonequilibrium chemically reacting effects may be important. The reference temperature method, commonly used to approximate the skin friction coefficient, was developed for the flow of a calorically perfect gas[3–8] and later for equilibrium dissociated air.[9–11] Reynolds analogy, used in conjunction with the skin friction coefficient to approximate the Stanton number (St), and hence the heat transfer, shows a similar development for a calorically perfect[8] and an equilibrium gas.[9–11] The purpose of the present study is to assess the effectiveness of these two engineering approximations for nonequilibrium flowfield conditions.

Theory

A nonequilibrium chemically reacting flow is one in which finite rate chemical reactions dictate the chemical composition of the gas. For high-temperature air, dissociation and recombination (and perhaps ionization) are important. In chemically reacting flows, heat transfer rates become a dominant aspect, and in nonequilibrium flows various physical properties such as enthalpy, specific heats, and internal energy are functions of not only the temperature and pressure, but also of the "history" of the flowfield (via the nonequilibrium chemical composition).

The reference temperature method allows for the computation of skin friction coefficients for compressible flow by using classical results of incompressible flow. For example, for incompressible flow over a flat plate

$$c_f = (0.664/\sqrt{Re_x}) \tag{1}$$

One way Eq. (1) can be "corrected" for compressibility effects is by using

$$c_f \approx (0.664\sqrt{C^*}/\sqrt{Re_x}) \tag{2}$$

where Sutherland's law was used for C^*, given by

$$C^* = \left(\frac{T^*}{T_e}\right)^{1/2} \frac{T_e + K}{T^* + K} \tag{3}$$

In Eq. (3), K is a constant that depends on the gas considered. For this study, a value of $K = 250°\text{R}$ is used for molecular oxygen. Two equations used for T^* were compared in the present study

$$\frac{T^*}{T_e} = 1 + 0.032M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right) \tag{4}$$

derived by Rubesin and Johnson,[3] and

$$\frac{T^*}{T_e} = 1.28 + 0.023M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right) \tag{5}$$

from Young and Janssen,[4] who found this form of the equation to be more accurate for a calorically perfect gas for $M > 5.6$. As will be shown, Eq. (5) gave better results under the conditions considered for this study.

Reynolds analogy is a proportionality between St and the skin friction coefficient c_f, and can be expressed in the following form for flow over a flat plate:

$$St = \tfrac{1}{2}c_f Pr_w^{-2/3} \tag{6}$$

As the following explains, a range of different values for Pr_w and c_f were considered in this study.

Method of Analysis

It is unlikely that the reference temperature method would be accurate for a multispecies air model with a complex geometry if it did not prove effective for a simple reacting gas model/geometry configuration. For this reason, a flow of a binary mixture of atomic and molecular oxygen over a 10-ft flat plate was considered. (This work is now being extended by Ott[12] using a seven species air model over cone-derived "waverider" configurations.)

The boundary-layer equations for a two-dimensional, laminar, nonequilibrium chemically reacting binary mixture of atoms and molecules were solved with an implicit finite-difference scheme. In the interest of space, the details of the governing equations, chemistry, boundary-layer characteristics, and numerical method can be found in Ref. 13. This approach is patterned after that of Blottner.[14,15] Therefore, no further details will be given here.

Initial conditions for the nonequilibrium calculation in the form of velocity and temperature profiles were obtained by solving the boundary layer equations for the flow of a calorically perfect gas. For the specific case of flow over a flat plate, these equations are indeterminate at the sharp leading edge ($\xi, x = 0$) so the solution was carried out at a distance of 0.008 ft from the leading edge. Calorically perfect initial profiles close to the leading edge were considered sufficient since the flow is essentially frozen in that region. No difference in the downstream results was observed when either calorically perfect or nonequilibrium initial profiles were used.

The boundary condition at the outer edge of the boundary layer was chosen to be freestream conditions taken from Ref. 16. A constant temperature wall boundary condition was used as well as the assumption of an equilibrium fully catalytic wall. (Note: these results are not necessarily applicable for a noncatalytic or a partially catalytic wall.) The wall temperature

Received April 16, 1993; revision received July 6, 1993; accepted for publication July 26, 1993.

*Graduate Student, Aerospace Engineering; currently Member of the Technical Staff, Computational Physics, Inc., 2750 Prosperity Ave. #600, Fairfax, VA 22031. Member AIAA.

†Professor, Aerospace Engineering. Fellow AIAA.

generally varied from case to case, and was set by assuming a wall-to-adiabatic wall enthalpy ratio as follows:

$$0.02 \leq (h_w/h_{aw}) \leq 1.0 \tag{7}$$

which gave a range of wall temperatures from around free-stream values to about 3600°R (2000 K), the usual upper limit applied to current and near future material temperature limits.

The thermodynamic properties of the binary mixture must be calculated for the individual species first and then combined to find mixture values. The specific heats at constant pressure included translational, rotational, and vibrational energies, but excluded electronic energy.

Mixture values were obtained by summing the individual contributions multiplied by their respective species concentrations. Enthalpies are given by similar relations with mixture values found in the same manner. The enthalpy of the atomic species contain the powerful heat of formation of oxygen.

Coefficients of viscosity and thermal conductivity were calculated using the Lennard-Jones potential as the model of the intermolecular force field where the viscosity and thermal conductivity are functions of collision integrals. Details of this analysis can be found in Ref. 17. Once transport properties for the individual species were found, Wilke's rule was used to obtain mixture values.

Results

A total of 95 different cases with a variation of Mach number, altitude, and wall-to-adiabatic wall enthalpy ratio as the parameters of this study were run. Only representative results are given in Table 1. In this table results are given from one specific altitude and Mach number for the spread of enthalpy ratios used for that Mach number (i.e., only Mach 5 results for 50,000 ft are given, although results for the other altitudes were also obtained—see Ref. 13). (Enthalpy ratios were chosen which gave wall temperature ratios within material limits as explained earlier.) The last two columns contain the percent mean relative errors between calculated skin friction coefficients from the exact boundary-layer finite difference solution and those generated by the reference temperature method. Note that a value in one of these last two columns, such as −2.015, indicates that the reference temperature method generates skin friction coefficients that were, on the average, 2.015% lower than exact values. Results using both Eqs. (4) and (5) are given.

For all 95 cases given in Ref. 13, the mean relative errors were generally appreciably lower using Eq. (5) rather than by using Eq. (4), and were all below 6.3%. Except for a few cases with the highest wall temperatures and some low altitude Mach 25 cases, the mean relative errors were within 5% and were generally below the exact values.

Thus, in spite of the presence of finite rate chemistry in the boundary layer and atom mass fractions at the wall that approached 20%, the reference temperature method proved effective and accurate.

Reynolds analogy was tested through evaluation of St from the finite difference boundary-layer results and compared with the values generated by Eq. (6). When using Reynolds analogy, one has a choice of values used for the skin friction coefficient and the wall Prandtl number. Since the main advantage of this engineering approximation is its ease of implementation, a constant wall Prandtl number was used to compare with exact Stanton number values. Two different constant wall Prandtl numbers were tested, the usual air value of 0.71 and a value of 0.75, which was used by van Driest[18] and which also gave the best overall results. This latter value was also close to the mean of the wall Prandtl numbers (0.7677) calculated by the boundary-layer solution for the cases considered. Both exact skin friction coefficients and those generated by using the reference temperature method with Eq. (5) were tested. Exact boundary-layer results for c_f were used in Reynolds analogy to give St; these values of St are compared with exact values of St obtained directly from the finite difference boundary-layer results. In addition, the reference temperature results for c_f are inserted in Eq. (6) to yield an approximate value of St, which is then compared with the St obtained from the finite difference boundary-layer solution. The four different combinations of c_f and Pr give the following functions:

$$St_1^* = \tfrac{1}{2}(c_f^*)_{YJ}(0.75)^{-2/3} \tag{8a}$$

$$St_2^* = \tfrac{1}{2}(c_f^*)_{YJ}(0.71)^{-2/3} \tag{8b}$$

$$St_3 = \tfrac{1}{2}c_f(0.75)^{-2/3} \tag{8c}$$

$$St_4 = \tfrac{1}{2}c_f(0.71)^{-2/3} \tag{8d}$$

Starred quantities indicate that reference temperature values were used, and the subscripts 1–4 are merely to identify the equation used in tabulated results.

Table 1 Skin friction coefficient percent relative errors

Case	h_w/h_{aw}	T_w, °R	$c_{A\,wall}$	Eq. (4)	Eq. (5)
M = 5, 50 kft	0.4	801	0.0	0.8977	0.5716
	0.7	1355	0.0	0.3148	0.0406
	1.0	1873	0.0	−0.2331	−0.4796
M = 10, 100 kft	0.1	720	0.0	−0.8721	1.843
	0.2	1385	0.0	1.536	0.7837
	0.4	2594	8.9×10^{-6}	−2.719	−0.8920
	0.55	3466	0.0018	−3.518	−1.939
M = 15, 150 kft	0.05	915	0.0	−5.335	−0.7098
	0.1	1736	0.0	−5.857	−1.639
	0.2	3252	0.0018	−6.673	−3.141
	0.25	3991	0.045	−5.952	−2.647
M = 20, 200 kft	0.025	738	0.0	−7.644	−2.015
	0.05	1415	0.0	−8.028	−2.692
	0.1	2651	1.1×10^{-5}	−8.452	−3.642
	0.15	3838	0.07	−7.135	−2.699
M = 25, 250 kft	0.02	771	0.0	−8.570	−2.498
	0.05	1806	0.0	−9.123	−3.447
	0.07	2447	1.1×10^{-5}	−9.286	−3.846
	0.09	3074	0.006	−9.288	−4.017

Table 2 Reynolds analogy percent relative errors in Stanton number

Case	h_w/h_{aw}	T_w	$c_{A\,wall}$	St_1^*	St_2^*	St_3	St_4
M = 5, 50 kft	0.4	801	0.0	1.461	5.200	0.886	4.620
	0.7	1355	0.0	3.961	7.830	3.950	7.818
M = 10, 100 kft	0.1	720	0.0	2.857	6.636	1.053	4.719
	0.2	1385	0.0	3.439	7.288	2.635	6.454
	0.4	2594	8.9×10^{-6}	3.145	6.984	4.074	7.947
	0.55	3466	0.0018	3.418	7.266	5.463	9.387
M = 15, 150 kft	0.05	915	0.0	1.942	5.736	2.584	6.377
	0.1	1736	0.0	1.850	5.641	3.532	7.385
	0.2	3252	0.0018	0.798	4.549	4.066	7.939
	0.25	3991	0.045	3.984	7.853	6.810	10.785
M = 20, 200 kft	0.025	738	0.0	2.015	5.811	3.916	7.783
	0.05	1415	0.0	1.578	5.358	4.328	8.211
	0.1	2651	1.1×10^{-5}	0.4660	4.205	4.263	8.143
	0.15	3838	0.07	3.157	6.996	6.016	9.961
M = 25, 250 kft	0.02	771	0.0	3.110	6.948	5.481	9.406
	0.05	1806	0.0	1.593	5.374	5.159	9.083
	0.07	2447	1.1×10^{-5}	0.8598	4.613	4.882	8.785
	0.09	3074	0.006	0.5162	4.257	4.780	8.680

Sample results are given in Table 2, with the last four columns giving the mean percent relative error over the entire plate for the four forms mentioned above.

As can be seen from this table, a wall Prandtl number of 0.75 gives better approximations to the Stanton number than a value of 0.71. Also notice that the percent mean relative error in Stanton number calculation using exact skin friction coefficients [Eqs. (8c) and (8d)] instead of the reference temperature generated skin friction coefficients [Eqs. (8a) and (8b)] are, in general, slightly worse, with the difference becoming more marked as the Mach number increases. This occurs because the reference temperature method generally underpredicts skin friction coefficients, and Reynolds analogy generally overpredicts; hence these two competing trends tend to cancel each other, producing better overall results. Results for Reynolds analogy do not exhibit the same Mach number trends found in the reference temperature results, but for calculations with $Pr_w = 0.75$ and the Young and Janssen T^* equation, the mean relative errors are all below 4%.

Conclusions

The reference temperature method is found to be a reasonably accurate method of predicting the skin friction with the presence of finite rate chemistry in the boundary layer and with an equilibrium fully catalytic wall. The equation developed by Young and Janssen was the most accurate of all the equations considered and gave mean relative errors below 6.3%. The exact boundary-layer results indicated that Reynolds analogy is reasonably valid for nonequilibrium chemically reacting boundary layers. Moreover, the use of the reference temperature method in conjunction with Reynolds analogy gave reasonable engineering predictions for St, giving mean relative errors below 4%.

References

[1]Anderson, J. D., Jr., and Lewis, M. J., "Hypersonic Waveriders—Where Do We Stand?," AIAA Paper 93-0399, Jan. 1993.

[2]Anderson, J. D., Jr., Lewis, M. J., Kothari, A. P., and Corda, S., "Hypersonic Waveriders for Planetary Atmospheres," *Journal of Spacecraft and Rockets*, Vol. 28, No. 4, 1991, pp. 401–410.

[3]Rubesin, H. W., and Johnson, H. A., "Critical Review of Skin Friction and Heat-Transfer Solutions of the Laminar Boundary Layer of a Flat-Plate," *Transactions of the American Society of Mechanical Engineers*, Vol. 71, May 1949, p. 383–388.

[4]Young, G. B. W., and Janssen, E., "The Compressible Boundary Layer," *Journal of the Aeronautical Sciences*, Vol. 19, April 1952, pp. 229–236, 288.

[5]Eckert, E. R. G., "Engineering Relations of Friction and Heat Transfer to Surfaces in High Velocity Flows," *Journal of the Aeronautical Sciences*, Vol. 22, No. 8, 1955, p. 585–587.

[6]Eckert, E. R. G., "Engineering Relations for Heat Transfer and Friction in High Velocity Laminar and Turbulent Boundary-Layer Flow over Surfaces with Constant Pressure and Temperature," *Transactions of the American Society of Mechanical Engineers*, Vol. 78, No. 6, 1956, p. 1273–1283.

[7]Sommer, C. C., and Short, B. J., "Free-Flight Measurements of Turbulent Boundary-Layer Skin Friction in the Presence of Severe Aerodynamic Heating at Mach Numbers from 2.8 to 7.0," NACA TN-3391, March 1955.

[8]White, F. M., *Viscous Fluid Flow*, McGraw-Hill, New York, 1974.

[9]Cohen, N. B., "Correlation Formulas and Tables of Density and Some Transport Properties of Equilibrium Dissociated Air for Use in the Boundary Layer Equations," NASA TN D-194, Feb. 1960.

[10]Cohen, N. B., "Boundary-Layer Similar Solutions and Correlation Equations for Laminar Heat Transfer Distribution in Equilibrium Air at Velocities up to 41,000 Feet Per Second," NASA TR R-118, 1961.

[11]Zoby, E. V., "Approximate Relations for Laminar Heat-Transfer and Shear Stress Functions in Equilibrium Dissociated Air," NASA TN D-4484, April 1968.

[12]Ott, J., private communication, Univ. of Maryland, College Park, MD, Jan. 1993.

[13]Debrestian, D. J., "A Numerical Investigation of the Applicability of the Reference Temperature Method and Reynolds Analogy to Nonequilibrium Chemically Reacting Boundary Layers," M.S. Thesis, Dept. of Aerospace Engineering, Univ. of Maryland, College Park, MD, May 1992.

[14]Blottner, F. G., "Nonequilibrium Laminar Boundary Layer Flow of a Binary Gas," General Electric TIS R63SD17, 1963; see also AIAA Preprint 63-443, 1963.

[15]Blottner, F. G., "Chemical Nonequilibrium Boundary Layers," AIAA Journal, Vol. 2, No. 2, 1964, p. 232–240.

[16]NOAA, NASA, USAF, *U.S. Standard Atmosphere 1976*, Oct. 1976.

[17]Anderson, J. D., Jr., *Hypersonic and High Temperature Gas Dynamics*, McGraw-Hill, New York, 1989.

[18]Van Driest, E. R., "Investigations of Laminar Boundary Layers in Compressible Fluids Using the Crocco Method," NACA TN-2597, 1952.